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ENERGY CONSERVING IMPACT ALGORITHM FOR GAIT SIMULATION WITH PERSISTENT CONTACT

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INTRODUCTION

Substantial ground reaction forces exceeding body weight occur at heel strike. These impulsive loadings are believed to be important etiological factors in joint degenerative processes (Radin *et al.*, 1991). Modelling of the gait cycle and the associated joint mechanics therefore requires an accurate approach to the heel strike impact problem.

Various methods exist within analytical dynamics for solving the collision problem. Contact and impact are characterized by complicated non-holonomic unilateral constraints. The classical formulations lack the ability to solve problems of persistent contact or simultaneous multibody impact as seen during gait. These methods assume the time interval of contact to be infinitesimal, the configuration of the system not to change and the external forces to be negligible. Most impact formulations, the most common of which is the Newton formulation, are energetically inconsistent (Stronge, 1990; Keller, 1986; Kane and Levinson, 1985) and none allows for tangential compliance. Additionally, these formulations quantify the impulse rather than the contact force itself.

The objective of this paper is to present an energetically consistent numerical penalty method in order to investigate the ground reaction forces during the brief period of impact during gait.

RIGID-BODY CONTACT LAWS

The solutions to a rigid-body contact problem are bounded by the conditions of impenetrability at the contacting interfaces and by the frictional conditions between the surfaces.

The rigid-body contact law gives a unilateral constraint of impenetrability on the gap distance, d_n. Additionally, the contact forces, F_n, must be compressive. A third condition of complementarity states that the gap distance and the normal force must be orthogonal. These conditions express compactly using subdifferential inclusions (Moreau, 1974):

$$F_n \in \partial\psi_{\mathbb{R}^+}(d_n) \qquad d_n \in \partial\psi_{\mathbb{R}^-}(f_n) \tag{1}$$

where $\psi_{\mathbb{R}^+}$ and $\psi_{\mathbb{R}^-}$ are the indicator functions of $\mathbb{R}^+$ and $\mathbb{R}^-$, respectively.

The Coulomb friction relations can also be summarized by subdifferential inclusions:

$$v_s \in \partial\psi_C(F_t) \qquad F_t \in \partial\psi_C^*(v_s) \tag{2}$$

where F_t is the tangential friction component, μ is the coefficent of static friction, $\bar{C}$ is the Coulomb friction cone and v_s is the relative slip velocity magnitude.

The nonsmooth character of these laws produces finite jumps in the velocities.

PENALTY CONTACT FORMULATION

We propose a numerical penalty formulation for contact, in which the impenetrability approximation of the rigid-body formulation is relaxed. We take the contact time to be finite and allow the system's configuration to change during the impact. The non-impulsive external forces, while small compared to impulsive forces, are included since they can have a significant effect for prolonged contact. With this formulation the problems of simultaneous multibody impact, persistent contact and impact with contact at another point are all handled identically. Tangential and normal compliance are both modelled.

This formulation eliminates the need for the coefficient of restitution. Instead, a material-dependent stiffness coefficient, ϵ, and a damping parameter, β, are used.

In the present model, we consider point contact with Coulomb friction. The calculations for the normal and tangential forces are markedly different than in the Newton formulation, but the Coulomb law of friction remains the same. The contact forces depend entirely on the configuration of the system and the state of slipping or sticking at a given time step. Unlike the Newton approach, this penalty formulation does not change the dimension of the configuration space during contact.

The contact forces from the penalty formulation are incorporated into the equations of motion via a generalized force term and take the following form:

$$F_n = -\tfrac{1}{\epsilon} d_n - \beta \dot{d}_n \tag{3}$$

$$F_t = -\frac{\dot{d}_t}{||\dot{d}_t||} \min(|-\tfrac{\mu}{\epsilon} d_n| , |-\tfrac{1}{\epsilon} d_t|) - \beta \dot{d}_t \tag{4}$$

where d_t is the tangential penetration distance and the dots indicate time derivatives.

NUMERICAL EXAMPLES

We use two distinct models—a double pendulum to look at the energetics of heel strike and a second model with persistent contact at the back leg to investigate the ground reaction forces (see Figure [1]).

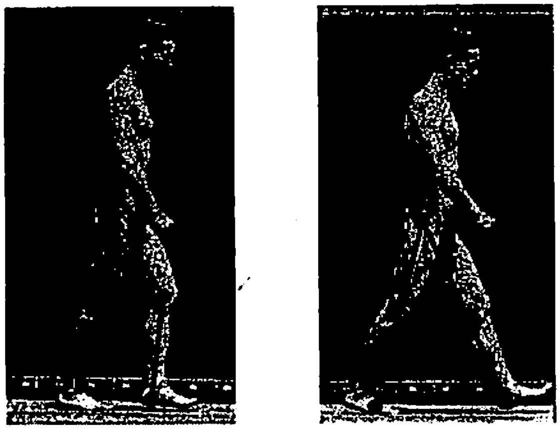


Figure 1: Gait models (from Muybridge (1981))

The simplified leg model illustrates the energetic consistency of the proposed method as well as the inconsistency of a Newton coefficient of restitution approach. For the double pendulum frictional collision with the ground Figure 2 shows the instantaneous energy gain of the Newton formulation. Even for coefficients of restitution smaller than one, Newton-based methods produce energy gains, whereas the penalty method is energetically consistent.

The simplified heel strike model with persistent contact and multiple contact points results in a normal-force profile qualitatively similar to that seen experimentally in gait studies (see Figure 3).

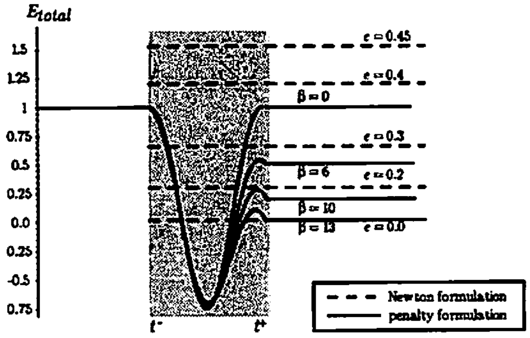
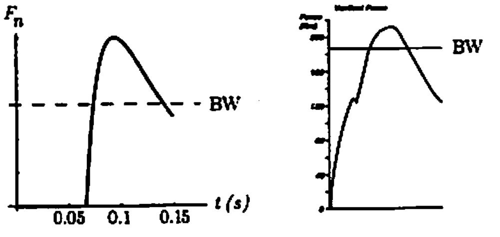


Figure 2: System energy over the impact



(a) numerical

(b) experimental (Perry, 1992)

Figure 3: Ground reaction forces

DISCUSSION

An energetically consistent impact model that examines impact forces and allows for multiple contact points was implemented using a penalty formulation. Contrary to the energetic inconsistencies of the Newton approaches and inability to handle persistent contact, this method accommodates well the heel strike impact problem. Consistent with experimental findings, we computed ground reaction forces greater than body weight during heel strike. This model is currently being extended to include soft tissue contributions and anatomical joints.

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