A VISCOELASTIC MODEL FOR FINITE DEFORMATION OF SOFT TISSUE

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INTRODUCTION

The ubiquitous presence of soft tissue in the human body has provided significant motivation for researchers to model its behavior. Specifically, soft tissue models for cartilage have been extensively developed. The biphasic (Mow et al., 1984) model has provided a foundation for the understanding of cartilage behavior, grounded in a solid-fluid interaction point-of-view. Phenomenological approaches have also been made, borrowing developments from viscoelasticity and applying those models to biomechanics. Woo et al. (1980) discussed the viscoelastic properties of cartilage in the context of a quasi-linear theory.

The finite deformation of soft tissue has been modeled with both the biphasic (Kwan et. al, 1990) and viscoelastic theories (Holzapfel, et al., 1996). Partly because of their complicated formulation, and partly because their developments in the context of the finite element method have only recently been made (Reese and Govindjee, 1998; Holzapfel, 1996), advanced nonlinear viscoelastic theories have been slow to make inroads into the biomechanics community.

The purpose of this contribution is to present a fully nonlinear viscoelastic model for soft tissue in finite deformation which not only maximizes the ability to capture creep behavior of biphasic models (Suh, 1995) and elaborate viscoelastic models (Holzapfel, 1996), but also minimizes the complexity required of the aforementioned developments.

FORMULATION

Viscoelastic models may be constructed either with evolution equations describing the time-dependence of internal rate variables, or by directly embedding an instantaneous rate-dependent response into the constitutive equation. While the former is more general, the latter offers simplicity in that viscous terms may simply be added to the stress response function. Accordingly, we propose an additive decomposition of the stress response function into elastic $\boldsymbol{S}^{elas}$ and viscous (inelastic) $\boldsymbol{S}^{visc}$ parts, written in the reference configuration,

$$\boldsymbol{S} = \boldsymbol{S}^{elas} + \boldsymbol{S}^{visc} \tag{1}$$

The rheological model for this construction is the Kevlin-Voigt viscoelastic solid, which we prescribe to have the form

$$\boldsymbol{S}(\boldsymbol{C}, \dot{\boldsymbol{C}}) = \underbrace{\lambda(J-1)\boldsymbol{C}^{-1} + \mu(\boldsymbol{C} - \boldsymbol{1})}_{\boldsymbol{S}^{elas}} + \underbrace{\eta\,\dot{\boldsymbol{C}}}_{\boldsymbol{S}^{visc}} \tag{2}$$

where $\boldsymbol{C}$ and $\dot{\boldsymbol{C}}$ are the right Cauchy-Green strain and strain rate tensors, respectively, and $\boldsymbol{S}$ is the second Piola-Kirchhoff stress tensor. In Eq. (2), λ and μ are Lamé constants and η is a viscous damping parameter. The particular form of $\boldsymbol{S}^{elas}$ offers robustness in compression, a characteristic not provided by Kirchhoff-St.Venant elastic material laws in finite deformation. Furthermore, $\boldsymbol{S}^{visc}$ adds a viscous contribution when stretch rates are encountered and provides a purely elastic response when $\eta = 0$.

RESULTS AND DISCUSSION

We first model a simple one-dimensional problem for confined compression of cartilage as discussed in Suh (1995) and compare the results obtained with the present material model to those from the standard biphasic theory.

Using $\lambda = 0.277$ MPa, $\mu = 0.130$ MPa, and $\eta = 10$ MPa-s, we obtained the cyclic compressive strain time histories as shown in Fig. 1. The strain history is dependent on the frequency of cyclic loading. Slower rates (0.001 Hz) invoke almost no viscous response whereas faster rates (0.01-0.1 Hz) cause the effects of viscosity to be seen.

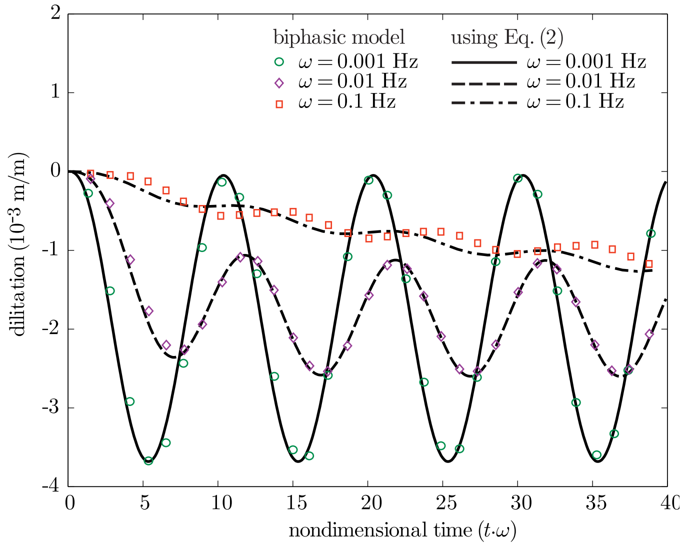

Figure 1: Comparison of strain response for cartilage in confined compression for the viscoelastic material model and the biphasic model.

In the second model we show the ability of the proposed model to capture the stretch time histories of more involved models for viscoelasticity. Using material properties $\lambda = 0.166$ MPa, $\mu = 0.015$ MPa, and $\eta = 1.06$ MPa-s, we obtained results for a creep extension test as shown in Fig. 2. Our simplified model produces the response of the more complicated model discussed by Holzapfel (1996) within a relative error of 20% and with good qualitative agreement. Moreover, both responses tend toward the theoretical maximum stretch of 5.0.

CONCLUSION

We have developed a simple viscoelastic material model suitable for the modelization of soft tissue. The additive decomposition of the stress response function is analogous to the infinestimal one-dimensional viscoelastic model of Kelvin-Voigt. In contrast, however, the present fully three-dimensional formulation satisfies the principle of objectivity and is both constitutively and geometrically nonlinear, suitable for finite deformations of soft tissue which may include softening/hardening, toe-in regions, and large rotations.

Many simple models for soft tissue are restricted to infinitesimal deformations. Some more general theories admit finite deformation, but do so at the cost of a computationally demanding formulation.

The proposed model effectively recreates the response histories of the biphasic model used in Suh (1995), and the creep tension test used in Holzapfel (1996). As such, this model offers an effective means to capture continuum responses of cartilage in a *simplified* framework, obviating the need for great sophistication in efforts to gain competitively similar results.

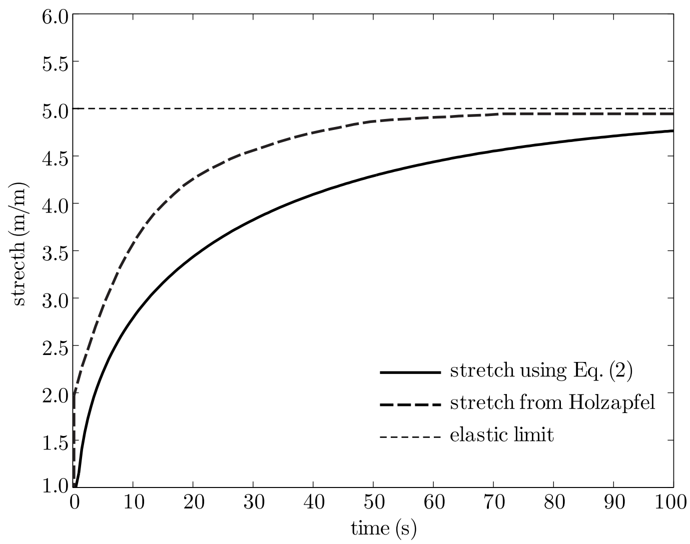

Figure 2: Creep test in tension – comparison of viscoelastic law to the model proposed by Holzapfel (1996).

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